

# Kinetic descriptions – a mathematical bridge to better understand the world

**Professor Eitan Tadmor** from the University of Maryland is director of KI-Net – a unique Mathematics research network that enables collaboration between participants in geographically distinct hubs and nodes. The network of 20+ nodes and more than 50 core participants is making significant contributions to the field of kinetic theory – mathematical description of small fluctuations with applications ranging from quantum chemistry to biology and social sciences.

**W**hen birds flock together, each individual bird interacts only with those closest to it; it aligns its velocity or its direction with an average velocity of its immediate neighbours. The adjustment that each bird makes involves only small local fluctuations. What we see from the ground, however, is quite different: we see a single mass that moves as one, creating mesmerising patterns. Kinetic descriptions is the mathematical tool which bridges the transition from modelling the small scale at the individual level of each bird, to the large scale realised at the group level of the flock.

#### STRENGTH IN NUMBERS

The idea that a sufficiently large number of small local fluctuations are combined to create a larger global impact, can be applied to many different areas. Take cells in the human body, for example. Each cell is acting only in response to its neighbouring cells; if you have a few cells in a petri dish, there is no large-scale outcome. However, the operation of millions of cells in the human body can self-organise to form organs as delicately complex as the eye or systems as wide-ranging as the blood vessel network. The term ‘many body problem’ refers to those configurations in which a global impact will emerge only if a sufficiently large number of interacting individuals, like cells or birds, are involved.

#### TRANSITIONING FROM MICRO TO MACRO

The original birthplace of kinetic theories is the emergence of properties of matter from local fluctuations at the molecular level. Take for example the air we breathe. Here, the microscopic fluctuations of air molecules, each

colliding with locally neighbouring molecules, take place on the atomistic scale. But when taking into account their huge number, the global effect of such collisions is realised on the macroscopic scale, as the temperature and density of the air. Kinetic description provides the link between these two scales. In the words of Professor Tadmor – “it is description across scales”. This paradigm of self-organisation is applicable in a broad range of different contexts, from individual molecules and birds to cars or even opinions which self-organise as part of a greater body of material, flock, traffic flow or consensus, through judicious interactions with local neighbours. It is this paradigm that Professor Tadmor has focused on in recent years.

#### THE RULES OF ENGAGEMENT

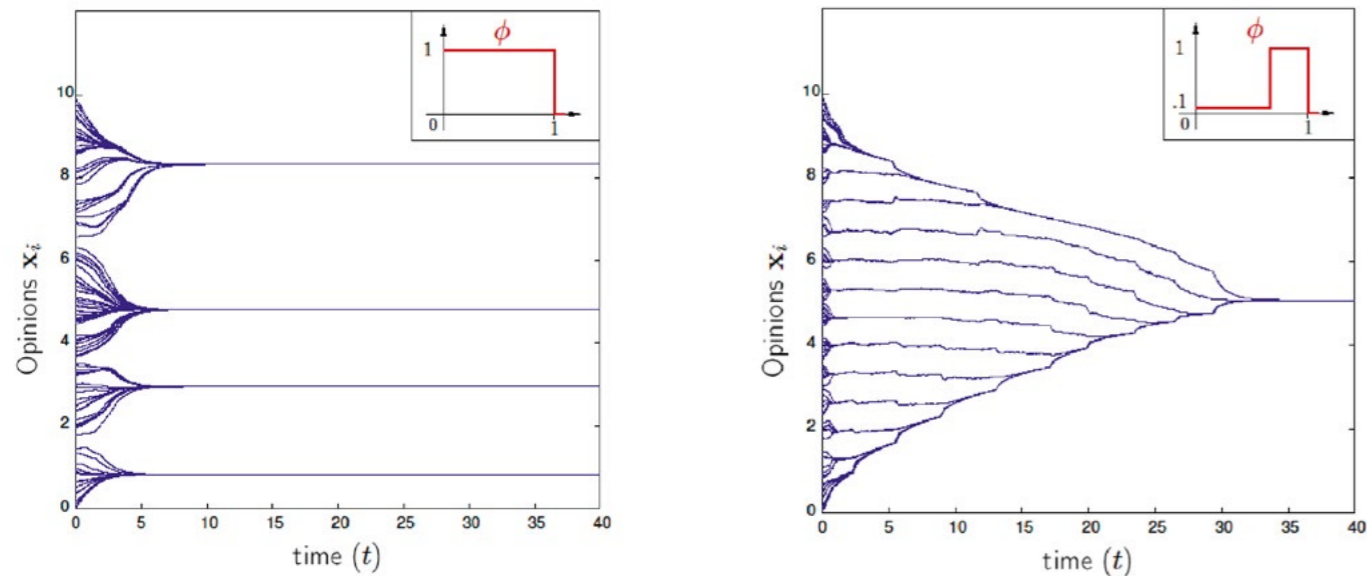
One area where the research focus of Professor Tadmor has led to fascinating, counter-intuitive outcomes is the propagation of opinions. Here, instead of molecules there are individual opinions; instead of collisions there is an exchange of ideas, where each individual is influenced by the opinion of its closest neighbours. Applying the methodology of kinetic descriptions to social sciences is still a fairly young discipline, but it can give invaluable insight into the way we organise ourselves. And it is here that Professor Tadmor emphasises the importance of rules of engagement: namely, how the local fluctuations due to neighbouring opinions have a global impact on the overall outcome in forming parties, reaching consensus, etc.

#### BIRDS OF A FEATHER FLOCK TOGETHER

Generally, we tend to gravitate towards those with similar opinions to our own, whether this is in the context of political parties or playground music preferences. In contrast, ▶

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**Figure 1** (Heterophilious vs. homophilious dynamics). Large-time behaviour of Hegselmann-Krause model with opinions distributed between 0 and 10 at  $t=0$ .

**Left figure:** homophilious dynamics. Interaction between any two opinions takes place only if the difference of their opinions is less than 1 (drawn by  $\phi$  on top right). It yields four distinct parties from time  $t=5$  thereafter.

**Right figure:** heterophilious dynamics. Same as before, but with LESS attention given to those which are 'close' in their opinion (if distance between two opinions is less than 0.7 then they take 1/10 of the importance of those who are farther away, with distance between 0.7 and 1 drawn by  $\phi$  on top right). Paying more attention to those who are different, leads to the emergence of consensus at time  $t=35$ .

those with significantly different opinions to ours have much less influence. In fact, we are more likely to dismiss their opinion entirely and stick to our own. Aligning with those you are already closest to is called 'homophilious' interaction.

**OPPOSITES ATTRACT**

Imagine, for a moment, a different scenario: rather than dismissing the opinions that are furthest from your own, you pay more attention to them, more than those opinions closest to yours. This is 'heterophilious' interaction. Contemplate the two different rules of engagement, homophilious and heterophilious interactions, and examine their global impact. Here comes the 'counterintuitive' part – according to Professor Tadmor, it is the heterophilious interaction of opinions which is more likely to produce consensus. But why? After all, the expectation is that interacting with those whose opinions correlate to yours is more likely to bring about agreement.

The answer lies in the communication among individual opinions. Homophilious interactions tend to cluster those that think alike together, and consequently, they tend to separate into different groups or parties of distinctly polarised opinions which lack the ability to influence each other. Communication therefore breaks down. However, placing a greater emphasis on alignment with those

further away is more likely to keep the lines of communication among individual opinions open, so that the many fluctuations of local compromises on the individual scale add up and, counterintuitively, consensus emerges on the large scale. As Professor Tadmor explains, while in the homophilious case a disconnect could arise between polarised groups, "heterophilious dynamics prevents the scenario of dysconnectivity".

**THE EMERGENCE OF CONSENSUS**

This emergence of consensus, as Professor Tadmor emphasises, is not a certainty but a 'more likely' possibility. In fact, the theory of kinetic descriptions has an essential probabilistic element. When considering the interaction of the 'many body problem' or engagement of many individual birds, cars or opinions, one has to contemplate many, many more possible interactions. Kinetic descriptions deal with the most likely outcome out of this huge ensemble of possible configurations. Kinetic description governs

the transition from the microscopic level to the human, macroscopic level, not in terms of certainty of the outcome but in terms of the probability of the most likely final outcome.

**KI-NET**

The theory of kinetic descriptions has a long history, branching out from James Clerk Maxwell's work on statistical mechanics in the mid-19th century. The body of work that has developed since then has been remarkably successful at predicting large-scale phenomena across natural, life, and more recently, social sciences. This is now being added to by the work at KI-Net. This research institute, headed by Professor Tadmor, is unique in its structure. Rather than being based at a single institution, its members are spread across sites, mainly in the US but also in Europe. Researchers outside of the network also collaborate on each project so the crowd of hundreds of scientists involved with KI-Net activities is even greater than the currently expanding 50 core participants.

**With applications ranging from propagation of opinions to tumour formation and traffic monitoring, the mathematical theory of kinetic descriptions encompasses a broad range of applications**



Faced with the complexities of directing a large network that is not based in a physical institute, Professor Tadmor and his IT team in the Center for Scientific Computation and Mathematical Modelling (CSCAMM) at the University of Maryland created an online platform which enables the network to run a decentralised yet synchronised series of activities. Participants can add information, set up requests and disseminate information about projects via the dedicated KI-Net online platform. At the same time, the three main hubs involved in the network (University of Maryland; University of Texas-Austin; University of Wisconsin-Madison) can then allocate finance in response. Crucially, however, the platform is also a base for KI-Net core participants to communicate, exchange knowledge, collaborate and develop links with other researchers.

Much like its research focus on kinetic descriptions, KI-Net itself is a platform that provides an additional level of communication between individual researchers, whose interactions promote the larger body of work on this area. Perhaps the most important product of KI-Net, says Professor Tadmor, is creating the sense of community. A key example is the network's focus on the support of junior researchers. The KI-Net annual Young Researchers Workshops (YRW) have become a key date in the calendar, expanding each year. Unlike a usual conference, the focus here is solely on junior, pre-tenure researchers who make up an exclusive group of participants and invited speakers. The KI-Net YRWs have proven enormously beneficial, not only in creating a community of talented junior researchers, but also to the institutions involved, looking to attract top talent.

**FLYING HIGH**

Professor Tadmor has had a distinguished career as a leading figure heading several research institutes (including Sackler's in Tel-Aviv University, IPAM at UCLA and CSCAMM in Maryland) of which KI-Net is the most recent. He highlights the importance of collaborative work, stating that he feels "privileged" when the work he does has an impact, and his ideas are well-received in the community. As the recipient of the 2015 Peter Henrici prize for "original contributions to applied analysis and numerical analysis", it is clear that his voice carries a considerable weight.

**UNIVERSAL LANGUAGE**

Professor Tadmor speaks with evident passion for the subject of Maths: "Maths is a language. There is something very warming and welcoming when you meet other colleagues across the world, and you feel almost at home because you know how to speak this universal language of Maths with its many dialects". With applications ranging from the propagation of opinions to tumour formation and traffic monitoring, the mathematical theory of kinetic descriptions encompasses a broad range of applications.

Professor Tadmor is convinced of the importance and value of continued work in the area of kinetic descriptions: "It took many years to develop this paradigm, but it is far from being complete. There are many questions which are yet to be addressed; and that's exactly what we do in the KI-Net – our KI-Net community is trying to build higher, reach farther and dig deeper in the theory of kinetic descriptions."

**Detail**



**RESEARCH OBJECTIVES**

Professor Tadmor has been involved in ground-breaking work on the theory and computation of differential equations which involve high-resolution schemes for shock waves and kinetic formulation of conservation laws. He introduced novel ideas of multi-scale hierarchical descriptions of images, and he is currently leading an interdisciplinary programme on self-organised dynamics with applications to flocking and propagation of opinions.

**FUNDING**

National Science Foundation (NSF)

**COLLABORATORS**

As the KI-Net Director leading its main hub at the University of Maryland, Professor Tadmor is collaborating with Professor Irene Gamba and Professor Shi Jin who, respectively, lead the two other KI-Net hubs at the University of Austin-Texas and University of Madison-Wisconsin.



**BIO**

Professor Tadmor received his BSc, MSc and PhD in Mathematical Sciences from Tel-Aviv University in 1973, 1975 and 1979, respectively. Following

his post-doc at Caltech, he returned to Tel-Aviv University, where he chaired the Department of Applied Mathematics. In 1995 he moved to UCLA where he co-founded the Institute for Pure and Applied Mathematics (IPAM), before joining the University of Maryland in 2002, heading its Center for Scientific Computation and Mathematical Modelling (CSCAMM), 2002–2016. This year, Prof Tadmor spends a sabbatical at ETH-Zurich as a Senior Fellow at the Institute for Theoretical Studies.

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