Dislocation avalanches:

An elegant analogy for metal deformation

When metals are placed under load, we have known for a long time that their constituent atoms will re-arrange themselves to produce strain. However, until recently, it remained a mystery to physicists exactly how a metal can apparently flow when strained yet remain solid. Professor Mick Brown, now an Emeritus Professor at Cambridge University, has devoted his research to finding out what is happening in deformed metals on a molecular scale. The analogy he uses is simple, yet remarkably effective.


Sand piles arrange themselves into a formation that minimises stress - the 'angle of repose'.

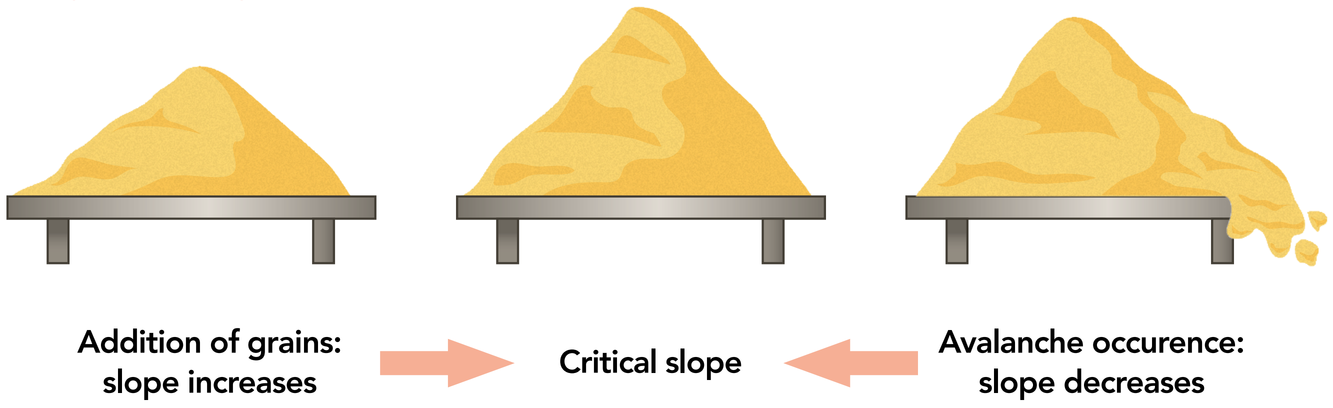
Solid materials can display a diverse range of behaviours when loads are applied to them. The bonds between the constituent atoms and molecules in the material will dictate how much the shape of the solid will deform, or 'strain', as well as how quickly the change will occur over time. Yet despite the fact that different solids will deform at different rates when a load is added, physicists can use universal laws to predict how deformations will progress over time, and for different loads. Two stages of the deformation process are particularly important. Initially, a deformation is 'elastic', meaning the solid will return to its original shape once the load is removed. However, once 'plastic' deformation is reached, the solid will be deformed permanently.

Professor Brown notes that these deformation properties have had widely-used practical applications for a long time. "Elastic bands stretch according to the load upon them, and then shrink back when the load is released," he explains. "However, all materials show an 'elastic limit' and under a large load deform permanently and then break. This property of 'plasticity' as opposed to 'elasticity' has been used for millennia to make coins when ductile metals are stamped to keep the monarch's head permanently visible."

THE PROBLEM WITH METALS

Deformation is easy to understand for many organic materials like rubber and plastic, but the process becomes more complex when metals are involved. Where intermolecular bonds in materials like rubber or plastic are weak, allowing them to deform easily, bonds between metallic atoms are far more rigid, making it difficult to imagine how metals can become deformed. "Metals are crystals, just like minerals, so how can they flow like a liquid, yet still keep their structure? The answer was given in the 1930s: defects called dislocations in the crystalline lattice of atoms move like rucks in a carpet and so allow the crystal planes to shear over one another," says Professor Brown.

THE SANDPILE MODEL



The sandpile model gives an example of self-organised criticality. As more sand is added, the slope increases in steepness until a critical point is reached. An avalanche then occurs which decreases the steepness of the slope. Image adapted from figure originally published in journal of Frontiers in systems Neuroscience: www.frontiersin.org/articles/10.3389/fnsys.2014.00166/full

Dislocations come in 'edge' and 'screw' forms and arrange themselves in tangles in which they block each other's movement. These arrangements of dislocations affect how metals are deformed on a macroscopic scale. In his research, Professor Brown has attempted to discover how dislocations affect deformations in metal over time. "In a metal under load, large numbers of dislocations flow in intermittent little jerks like grains of sand, impeding one another's progress, making it progressively harder for the metal to change its shape," he continues. "Many unsuccessful and controversial attempts were made to find the relationship between the load and the permanent deformation, something of obvious engineering utility."

VISUALISING A SAND PILE

Comparing dislocations to sand grains has allowed Professor Brown to devise a remarkably simple analogy to visualise how metal deformations are affected by dislocations. He proposes that a metal as it is being strained resembles sand being continuously poured into a pile. Initially, friction forces between the sand grains will resist gravity; holding each grain in place to maintain the structure of the pile. However, pouring more sand onto the pile will cause its sides to become steeper and steeper, making it increasingly difficult for the friction between the grains to maintain the pile's structure. Eventually, many grains will give out at once; slipping together in sudden avalanches. In a metal, the dislocations slip together to produce bursts of strain.

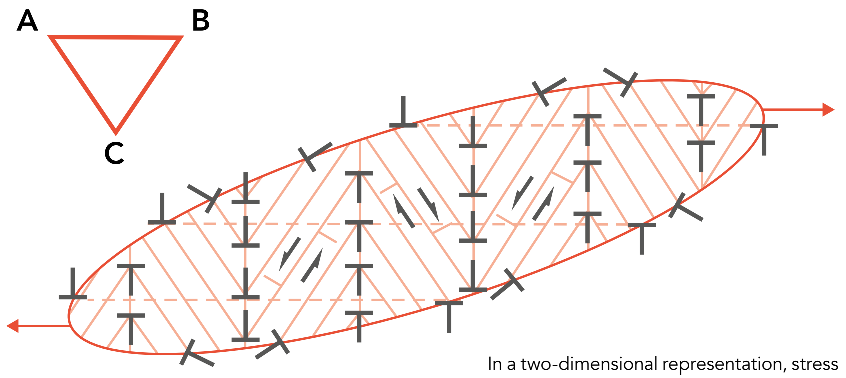
> ***In a metal under load, large numbers of dislocations flow like grains of sand, impeding one another's progress.***

As Professor Brown explains, there is consistent mathematics underlying how these avalanches come to a rest. "As sand is added to the pile, avalanches of sand course down the sloping sides until they come to a critical angle which they maintain – the angle you see over and over again on the beach, sometimes called the 'angle of repose'," he says. "At the angle of repose, each avalanche is brought to a halt, and then another one starts out and comes to the same fate." This behaviour may be fairly simple, but the mathematics dictating sand avalanche formation are remarkably similar to the laws underlying deformations in metals.

SELF-ORGANISED CRITICALITY

Like a sand pile, many systems in physics will suddenly rearrange themselves into new formations to respond to the forces upon them. "This behaviour is termed 'self-organised-criticality'," says Professor Brown. "It is the property of a complex particulate system under stress which enables a kind of steady state, where the system is apparently in repose although

ELLIPSOIDAL SLIP BANDS



In a two-dimensional representation, stress causes plastic shear of metal along the line AB. The shear is confined to an ellipsoid. Any slight tilt of the ellipsoid elongates it, and so excites shear along the lines AC and CB, which blocks further shear inside the ellipsoid. The packing of the tilted ellipsoids determines the increment of plastic strain in the metal. Edge dislocations are represented by the 'T' symbols.

Avalanches have no intrinsic size, so the laws they obey must be very simple; based on a power law.

it is constantly suffering avalanches. The state is 'critical' in that all the sand at the surface is in a state of incipient flow so that a small increment of a few grains of sand can cause an avalanche, and it is 'self-organised' because there is no-one outside the system controlling the flow."

Previously, physicists have modelled the flow of metallic atoms undergoing plastic deformation as a gradual, continuous process – an inaccurate representation of what is really happening. Professor Brown believes the physics underlying self-organised criticality solves this issue. The analogy replaces the continuous flow of atoms with sudden, randomly-occurring avalanches of dislocations, allowing models to predict metal deformations far more accurately.

AVALANCHES OF DISLOCATIONS

To understand the dynamics underlying avalanches of metal dislocations, Professor Brown directly relates the properties of sand piles to metals. "In the metal, the dislocations take the place of the sand grains, and the avalanches are called 'slip bands' or 'bursts of slip'," he explains. For dislocations, the physics underlying stresses give slip bands distinctive shapes and sizes, which can be approximated using mathematics.

"Because the avalanches are formed of dislocations moving together, experiencing the same stress so they travel at the same speed, the avalanches must have an ellipsoidal shape, and the packing of the ellipsoids plays a role in the arrangement of avalanches, each avalanche composed of dislocations," continues Professor Brown. "In a sand pile, just as the avalanches range in size from just a few grains of sand to almost the whole side of a pile, so in a metal they range from a few dislocations to a very large number." This analogy made it remarkably easy for Professor Brown to visualise patterns in dislocation slip band formation. To model metal deformations, the next step was to construct mathematical laws to describe how dislocation slip bands form and progress.

CONSTRUCTING A POWER LAW

Just as in Professor Brown's sand pile analogy, many systems in physics obey robust mathematical laws. By observing their properties through experiment, many physicists aim to construct equations which approximate these laws as accurately as possible. Professor Brown believes this process can be applied when predicting the sizes of dislocation slip bands. "Avalanches have no intrinsic size, so the laws they obey must be very simple; based on a power law," he says.

Power laws are a simple mathematical concept, describing the relationship between two quantities, where a fractional change in one results in the proportional fractional change of the other. Professor Brown describes how power laws in elastic systems are based on a power law named their 'elastic modulus', accounting for the quantities of load and extension. "In an elastic band, the load is proportional to the extension, often called Hooke's law. The properties of the rubber play an important role so the elastic modulus can in principle be calculated," he continues.


Coins exploit the property of 'plasticity' to keep the monarch's head permanently visible.

Essentially, the power laws express the fact that the frequency of occurrence of a large avalanche is small, but the frequency of small ones is very large, rather like earthquakes. It is remarkable to be able to calculate the power laws without knowing the complex processes underlying the formation of the avalanches.

When considering the more complex case of metal deformation, the sand pile analogy allows relevant power laws to be calculated. The mathematics Professor Brown describes bases deformation on stacks of elliptical avalanches of mathematically varying sizes. "In an ideal plastic metal crystal, the change in load is proportional to the change in plastic strain, which is the simplest possible power law," he explains. "There is a 'modulus' which is calculable from the way ellipsoids can be stacked. In fact, with this understanding it is possible to calculate several power laws obeyed by the distribution of avalanches, and to calculate what takes the place of the angle of repose: the slope of the graph of stress plotted against plastic strain."

EXPLAINING METAL PROPERTIES

Now retired, Professor Brown's research has contributed significantly to a robust mathematical description of metal deformation. This understanding has allowed him to explore several fundamental properties of metals. "Other 'emergent properties' of the metal can be rationalised, such as its ultimate tensile strength and its behaviour in repeated back-and-forth deformation: fatigue," he concludes. His research has allowed for a greater understanding of these properties on a molecular scale. The knowledge Professor Brown has gained could become remarkably useful for engineers; potentially helping them to construct new materials for use in industry and our everyday lives.

Behind the Research

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Research Objectives

Professor Brown's current work focuses on power laws in plastic flow.

Detail

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Prof L. M. Brown B.A.Sc. (Tor), Ph.D. (B'ham), M.A., Sc. D. (Cantab), F. Inst. P., F. I. M., Hon. F. R.M.S., F.R.S. is a Canadian by birth whose career has been almost entirely in England. He is now a Founding Fellow of Robinson College and a Professor Emeritus at the Cavendish Laboratory. He has published many papers in solid state physics and electron microscopy, particularly on work-hardening and fatigue of metals. Using electron energy loss spectrometry he and his students established the platelet structure of nitrogen contained in diamond. More recently, with Ondrej Krivanek he pioneered the design and installation of a practical corrector for spherical aberration in scanning transmission electron microscopes, leading to the establishment of the SuperSTEM facility in Daresbury, Cheshire. His most recent work has been to elucidate the mechanisms of crack initiation and the nature of the fatigue limit in ductile metals; also to model uniaxial deformation in the light of self-organised criticality, thereby deriving the various observed power laws relating mechanical properties.

Collaborators

Professor Brown would like to acknowledge his late great supervisor Prof Jock Eshelby, whose theorem on ellipsoidal inclusions plays such a pivotal role; and Prof. Sir Peter Hirsch, whose persistent studies of work-hardening and whose critical comments have been so helpful.

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Personal Response

What moments of your long career have you found the most enjoyable or satisfying?

" It has been deeply satisfying to develop these ideas with talented young research students at the Cavendish Laboratory. "