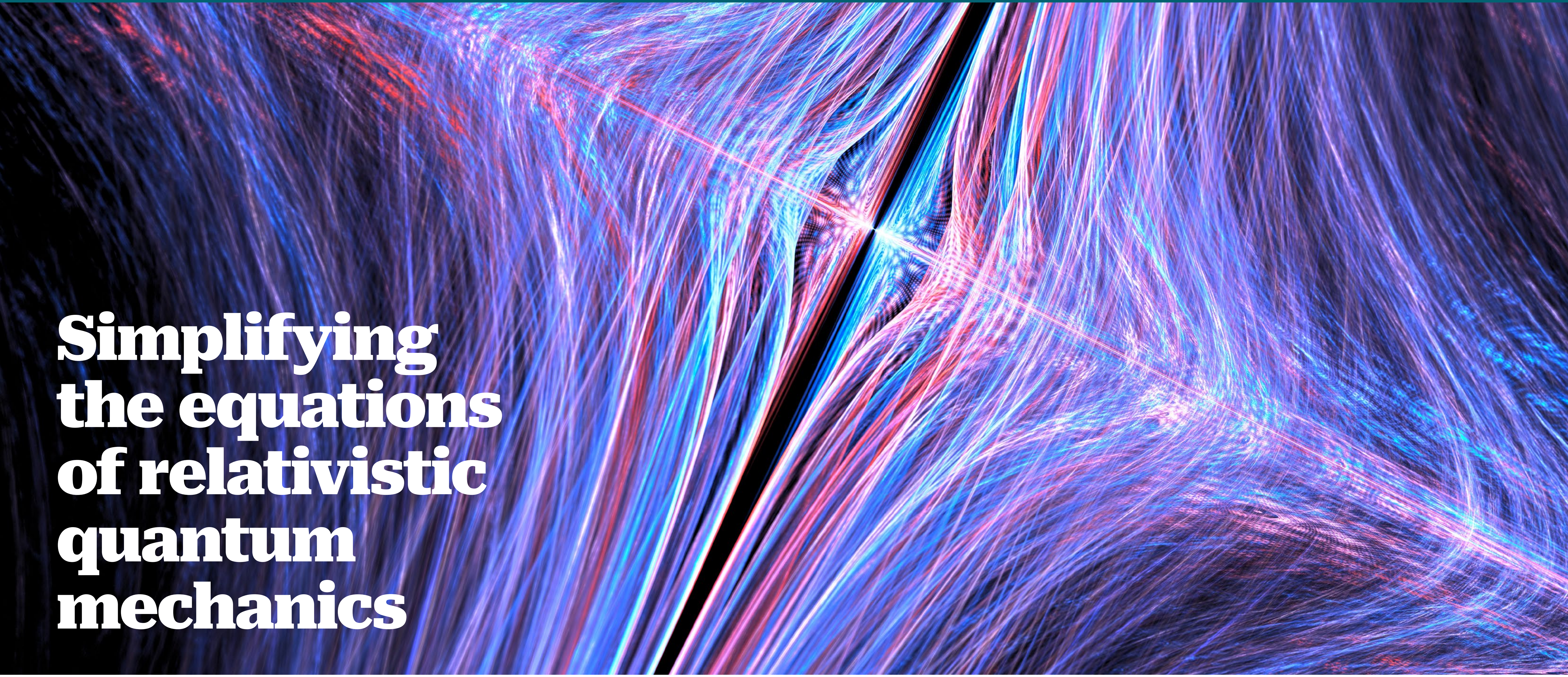


Simplifying the equations of relativistic quantum mechanics

- The equations of relativistic quantum mechanics represent our most up-to-date understanding of the quantum world, but they are too complex to be taught to undergraduate physics students.
- Dr Luis Grave de Peralta at Texas Tech University, USA, introduces a new set of equations which express the principles of relativistic quantum mechanics in far simpler terms.
- His team's work may also lead to fresh perspectives on some of the most long-standing mysteries in physics.

In the early 20th century, the groundbreaking theories of quantum mechanics and special relativity emerged at roughly the same time. At first, both theories appeared to be entirely incompatible with each other. While the quantum equations written out by Erwin Schrödinger and Wolfgang Pauli dealt with the behaviours of electrons, atoms, and molecules moving at speeds encountered in classical physics, the rules of special relativity first imagined by Albert Einstein described the behaviour of matter moving at close to the speed of light.

Initially, this apparent clash of theories presented a crisis in physics. However, in the late 1920s, further theoretical work carried out by physicists including Paul Dirac, Oskar Klein, and Walter Gordon culminated in a new set of equations, which proved definitively that quantum mechanics and special relativity are compatible with each other after all.

An educational challenge

Yet with this groundbreaking discovery came an entirely new problem. In adding a new layer of complexity to the equations of non-relativistic quantum mechanics, the more complete theories of relativistic quantum mechanics became far less accessible to physics students first being introduced to the strange world of quantum mechanics.

'Current textbooks of non-relativistic quantum mechanics are based on the use of the Schrödinger and Pauli equations', Dr Luis Grave de Peralta of Texas Tech University, USA, explains. 'In contrast, relativistic quantum mechanics requires use of the Klein-Gordon and Dirac equations, which demand more advanced mathematical skills to solve'. Ultimately, this means that with the exception of a few postgradate students who have chosen to study quantum mechanics in more detail, physics students today will still only encounter outdated descriptions of quantum mechanics – even almost a century after Dirac, Klein, and Gordon first wrote their equations. Considering the latest generation of students includes the researchers who will one day go on to advance existing theories further, Grave de Peralta believes the problem may be slowing the pace of cutting-edge research.

Simplifying the equations

In his research, Grave de Peralta aims to show how the complexity of the Dirac and Klein-Gordon equations may not be necessary to encapsulate the defining principles of relativistic quantum mechanics.

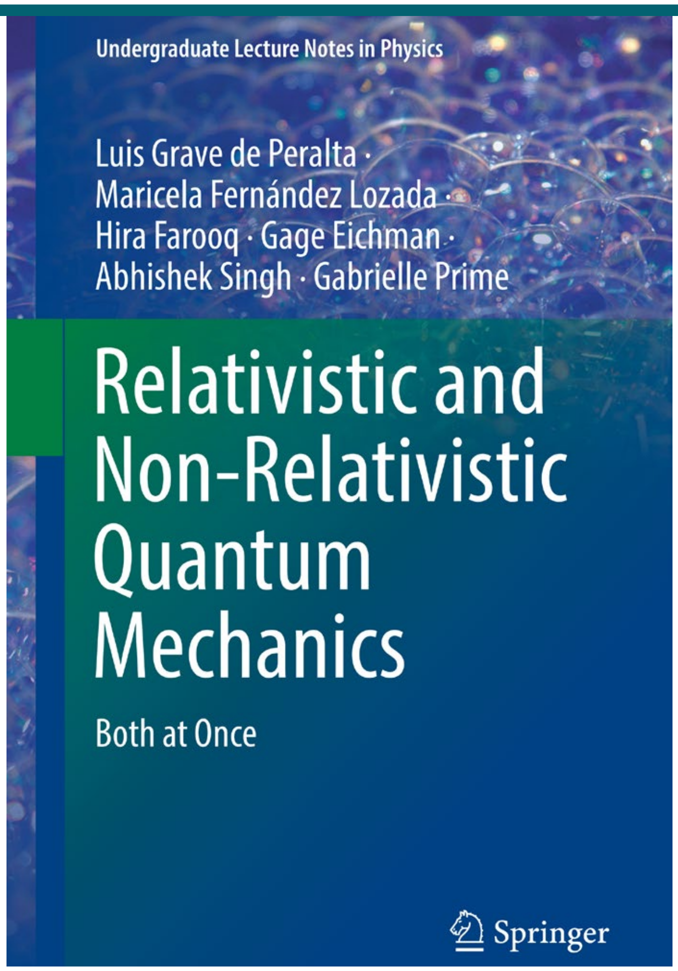
Through a series of recent studies, Grave de Peralta, together with colleagues Luis Poveda at CEFET-MG in Brazil, and Bill Poirier, also at Texas Tech University, present their 'Poveda-Poirier-Grave de Peralta' (PPGP) equations: which can express the relativistic behaviours of quantum particles in far simpler terms.

'We found that there are pairs of PPGP equations similar to the Schrödinger equation and the Pauli equation, whose solutions are equal to the solutions of the Klein-Gordon equation and the Dirac equation, respectively', Grave de Peralta says.

Relativistic quantum mechanics for students

Having established their theory, the researchers have now released

Relativistic quantum mechanics became far less accessible to physics students first being introduced to the strange world of quantum mechanics.



a new book entitled '*Relativistic and non-relativistic quantum mechanics: Both at once*', which uses the PPGP equations as a basis for developing students' knowledge of quantum mechanics in its most up-to-date form.

Beyond its promising applications in teaching, the book could even be used to examine the fundamental nature of the universe.

'In this book, we show that it is possible and convenient to use the PPGP equations for simultaneously teaching relativistic and non-relativistic quantum mechanics to undergraduate students', Grave de Peralta describes.

Grave de Peralta now hopes that his team's book could become a valuable resource for university courses – but also believes its potential could stretch far beyond its promising applications in teaching and could even be used to examine the fundamental nature of the universe from a fresh perspective.

The matter–antimatter imbalance

Among the lines of research Grave de Peralta's team consider is the apparent imbalance between matter and antimatter in the universe today. A particle of matter has the same mass but the opposite charge to its antimatter counterpart, as well as several differences in other quantum properties. When the two particles meet, they will completely annihilate each other.

According to physicists' existing theories, matter and antimatter should have been created in equal quantities during the Big Bang. Yet through their observations, astronomers can clearly see that the amount of antimatter in the universe today is vastly outweighed by the regular matter composing stars, planets, asteroids, and ourselves.

This imbalance has now become one of the greatest mysteries in modern cosmology and particle physics. While a wide array of theories have now been put forward to explain it, none of them have ever been proven definitively in experiments.

Providing a fresh perspective

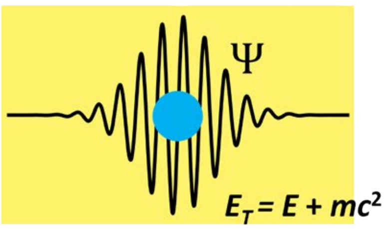
Grave de Peralta and his colleagues now hope that the simplified equations detailed in their book could finally help to bring an end to this deadlock. 'We propose a possible explanation for the observed asymmetry between matter and antimatter in the universe, which current physical theories cannot explain', Grave de Peralta says. 'We show that the relative simplicity of the PPGP equations allows us to explain important unsolved physical problems.'

Ultimately, in rewriting an important set of equations which have now been expressed in the same, deeply complex forms for close to a century, the team hopes that their PPGP equations could provide new perspectives on these problems, while perhaps bringing their long-awaited solutions a step closer to reality.

There are no waves associated to a classical particle: 

But there are two waves associated to a relativistic quantum particle:

The analytic expression of these waves are obtained solving the PPGP equations:



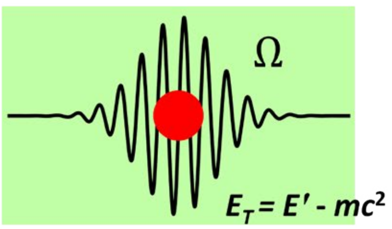
Schrödinger-like PPGP equation for spin-0 particles:

$$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2\mu(r)}\nabla^2\Psi + V(r)\Psi, \quad \text{with} \quad \mu(r) = \left[1 + \frac{E - V(r)}{2mc^2}\right]m.$$

Relativistic effective mass

Pauli-like PPGP equation for spin-1/2 particles:

$$i\hbar\frac{\partial}{\partial t}\Psi = \left[\hat{\sigma}\,.\left(\hat{p} - \frac{e}{c}A\right)\right]\left[\frac{1}{2\mu(r)}\right]\left[\hat{\sigma}\,.\left(\hat{p} - \frac{e}{c}A\right)\right]\Psi + eA_o\Psi.$$



Complementary Schrödinger-like PPGP equation for spin-0 particles:

$$i\hbar\frac{\partial}{\partial t}\Omega = -\frac{\hbar^2}{2\mu'(r)}\nabla^2\Omega + V(r)\Omega, \quad \text{with} \quad \mu'(r) = \left[-1 + \frac{E' - V(r)}{2mc^2}\right]m.$$

Relativistic effective mass

Complementary Pauli-like PPGP equation for spin-1/2 particles:

$$i\hbar\frac{\partial}{\partial t}\Omega = \left[\hat{\sigma}\,.\left(\hat{p} - \frac{e}{c}A\right)\right]\left[\frac{1}{2\mu'(r)}\right]\left[\hat{\sigma}\,.\left(\hat{p} - \frac{e}{c}A\right)\right]\Omega + eA_o\Omega.$$

There is a wave associated with a non-relativistic quantum particle but there are two waves associated with a relativistic quantum particle.

Personal response

Why have such complex expressions of relativistic quantum mechanics endured for so long?

This question came to my mind when I encountered the equation that now we call the Grave de Peralta (GP) equation for the first time. The GP equation is a particular case of the Schrödinger-like PPGP equation that occurs if the effective relativistic mass of the quantum particle is constant and positive.

I was greatly surprised to discover that I could publish this [De Peralta, LG, (2020) Did Schrödinger have other options? *European Journal of Physics*]. However, the GP equation is only a simple approximation to the exact PPGP equations. Probably, if somebody had already encountered the GP equation, the fact that the GP equation is only an approximation could be a little bit disappointing. Why had nobody found and used the PPGP equations before? This also is almost unbelievable!

Once I commented to Bill Poirier that my advantage was that I was not formed as a theoretical physicist; therefore, my mind is less constrained than the mind of somebody that only learned about the Klein-Gordon and Dirac equations. Nevertheless, Poveda and Poirier are exceptionally bright theoretical physicists. They complemented me very well. Our collaboration was an ideal combination of well-educated and free thinker physicists. It was the right combination for this task.

The key for loving the PPGP equations is to realise that, like the mass, the effective relativistic mass of a relativistic quantum particle does not depend on the spin of the particle. It has the same expression in the Schrödinger-like and the Pauli-like PPGP equations. In addition, you can obtain the analytic expression of the effective relativistic mass directly from the special theory of relativity by using simple arguments taken from the Dirac's Sea concept (see chapter 3 of the book).

How easily do you think university courses could adapt to using PPGP equations?

The changes are easy to implement. For example, our book is a first attempt to implement those changes. I have already taught quantum mechanics for undergraduate students twice following this approach. It works, but it could be improved. Nevertheless, I am sure there will be some opposition to introduce the required changes in well-established courses of quantum mechanics. I strongly believe that there are practical advantages associated with reaching a larger audience when talking about relativistic quantum mechanics. These advantages outweigh the cost related with modifying the current introductory quantum mechanics textbooks. I also believe this task requires young champions who will lead the second century of research on relativistic quantum mechanics. I wish and hope several of our readers will become the required champions.

Could the PPGP equations be applied and bring clarity to other solved or unsolved mysteries in physics?

The answer is yes. The PPGP equations can be used to explain and bring a new or a different perspective to several observable phenomena that can only be explained by combining the special theory of relativity and quantum mechanics. The list of these phenomena includes but are not limited to the following topics (see the cartoons in www.youtube.com/@luisgravedeperalta):

- Number of chemical elements in the Periodic Table. Why do only a hundred of chemical elements exist? Why not thousands of different chemical elements?
- Formation of black holes.
- Possible existence of primordial black holes (see chapter 8 of the book).
- The Planck's mass is the frontier between the quantum and the classical world. This hypothesis was first proposed by Roger Penrose, Nobel Prize in Physics.
- Why has no one ever observed Schrödinger's cat?
- Observed asymmetry between matter and antimatter.
- Possible existence of primordial electrical sinks (see chapter 8 of the book).

Details

e: luis.grave-de-peralta@ttu.edu
@luisgravedeperalta
w: Mecánica Cuántica Relativista y No Relativista: las dos a la vez: Parte I: Estados estacionarios by Luis Grave de Peralta | BookBaby Bookshop
w: Relativistic and Non-Relativistic Quantum Mechanics: Both at Once | SpringerLink

Collaborators

Coauthors: Maricela Fernández Lozada, Hira Farooq, Gage Eichman, Abhishek Singh, Gabrielle Prime.
Bill Poirier, Luis A Poveda, Arquimedes Ruiz Columbié.

Bio

Drs Luis Grave de Peralta and Hira Farooq, and undergraduate students, Gage Eichman, Abhishek Singh, and Gabrielle Prime are based at the Department of Physics and Astronomy, Texas Tech University, USA. Dr Maricela Fernández Lozada is a Cuban-Mexican theoretical physicist with many years of experience teaching quantum mechanics.

Further reading

- De Peralta, LG, Webb, K C, Farooq, H, (2022) A pedagogical approach to relativity effects in quantum mechanics, *European Journal of Physics 43*(4), 045402.
- De Peralta, LG, (2020) Did Schrödinger have other options? *European Journal of Physics 41*(6), 065404.
- De Peralta, LG, Poveda, LA, Poirier, B, (2021) Making relativistic quantum mechanics simple, *European Journal of Physics 42*(5), 055404.
- De Peralta, LG, Fernández Lozada, M, Farooq, H, Eichman, G, Singh, A, Prime, G, (2023) *Mecánica cuántica relativista y no relativista: las dos a la ves. Parte I: Estados estacionarios*.
- De Peralta, LG, Fernández Lozada, M, Farooq, H, Eichman, G, Singh, A, Prime, G, (2023) *Relativistic and non-relativistic quantum mechanics: Both at the same time.* English Edition Springer Nature, Undergraduate Lecture Notes in Physics, ISBN 978-3-031-37073-1.