String theory for builders

New mathematical perspectives on algorithms and complexity in quantum physics

- Many physicists hope that the long-awaited unification of all the fundamental forces will emerge from string theory.
- As described in accordance with quantum mechanics and relativity, the vibrations of elementary strings naturally include the quanta of gravity and elementary particles similar to those in the real world.
- Duality is a fascinating phenomenon in mathematical physics, with an incarnation in string theory, called gauge-string duality.
- Through a new research programme, an international team led by Dr Sanjaye Ramgoolam at Queen Mary University of London employs simple string theoretic quantum models, informed by ideas of duality, as a tool for answering questions on symmetry and complexity at the interface of mathematics and computer science.
- They hope that through string theory models, a deeper mathematical understanding of complexity in quantum physics can provide fresh perspectives on the relation between quantum physics and string theory, which will inform new approaches to the challenges of string theory.

Finding a unified description of the four fundamental forces of nature is a long-standing goal of theoretical physics. Three of these forces – the electromagnetic force, the weak nuclear force, and the strong nuclear force – shape our understanding of the microscopic world of atoms, nuclei, quarks, and leptons. They can be described using the mathematical language of quantum field theory, which is used to formulate the remarkably successful Standard Model of particle physics.

The fourth force is gravity. Described by Einstein's general theory of relativity, it shapes our understanding of the motions of planets, stars, galaxies, and the entire history of the universe. By unifying our understanding of the microscopic and the macroscopic through a quantum description of the four forces, researchers are now hoping to extend the reach of our physical theories into the very earliest moments of existence of the universe, and the latest stages of evolution of black holes.

String theory and the unification of forces

Currently, one of the most promising approaches to the unification of all forces lies with string theory. This intriguing branch of theoretical physics suggests that the fundamental particles included in the Standard Model are not really particles after all but arise from the vibrations of one-dimensional strings. Strikingly, these vibrations naturally include gravitons – the quantum particles arising from the gravitational fields in Einstein's theory of general relativity.

Among the known solutions of string theory are universes which include gravity, along with particles much like those in the Standard Model. However, the simplest solutions of string theory have ten dimensions of spacetime, far more than the four dimensions of our own universe. Narrowing this down to a four-dimensional universe involves navigating a vast and complex landscape of possibilities, where the rules of the journey are not yet understood.

For present-day theoretical physicists, understanding these rules and managing the complexity of the landscape is the challenge they have to face, to explain the nature of particle physics and quantum gravity in the real world.

Gauge-string duality and mathematical model universes

Despite these challenges, the landscape of possible universes in string theory also offers some unexpected insights into quantum gravity. One study in the early 1990s, for example, identified certain mathematical model universes within string theory which exhibited a striking 'duality' with ordinary quantum theories, with no explicit strings or gravity [10]. This is a type of duality known as 'gauge-string' duality.

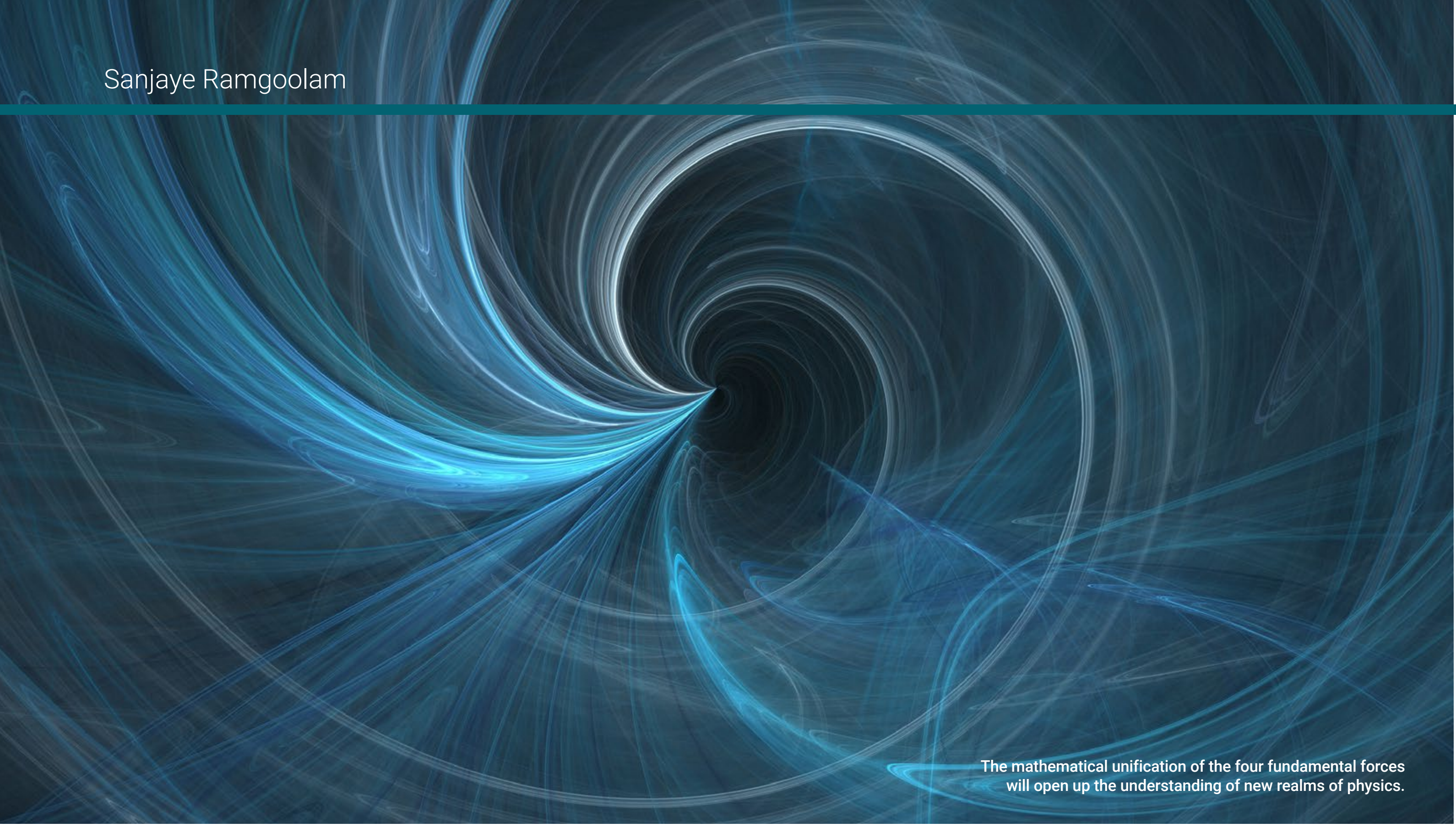
In this context, duality refers to a significant similarity and often, an exact but non-obvious equivalence between different looking mathematical models of physics. Many examples of this phenomenon are known in mathematical physics: for example, there is an equivalence known as S-duality which exchanges electric and magnetic particles in certain four-dimensional quantum field theories [7]. In 1997, Juan Maldacena discovered a phenomenon named 'the AdS/CFT correspondence' (anti-de Sitter/conformal field theory correspondence) where complex models of string theory, including those describing ten dimensional universes, exhibit a remarkable new example of 'gauge-string' duality [8]. When described using this type of duality, these ten-dimensional stringy universes are equivalent to four-dimensional quantum field theories.

> **Currently, perhaps one of the most promising approaches to the unification of all forces lies with string theory.**

Gauge-string duality has remained a central subject of study in string theory for the last 25 years. During this time, there has been significant progress in understanding the mathematical mechanisms of duality and its new physical applications, such as in heavy ion collisions.

String theory as a tool for studying the mathematics of algorithms and complexity

In a series of recent papers, Dr Sanjaye Ramgoolam of Queen Mary University of London and a team of international collaborators started a research programme to take a fresh look at simple mathematical models of string theory and duality [1–6] with a view to wider



The mathematical unification of the four fundamental forces will open up the understanding of new realms of physics.

In their papers, Dr Ramgoolam and Dr Ben Geloun developed quantum mechanical models where the quantum states are superpositions of ribbon graphs.

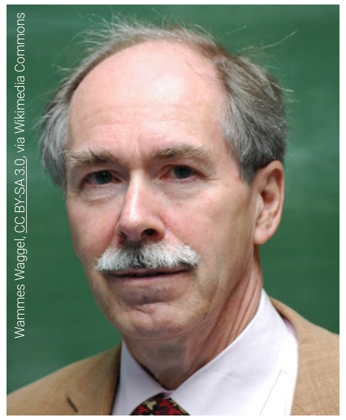
applications beyond string theory. Their approach uses these models as tools for studying questions about complexity and algorithms which arise at the interface of mathematics and computer science.

In particular, they focus on algorithms for mathematical objects called 'finite groups'. Finite groups are collections of objects subject to certain rules of composition which have applications in the description of symmetries of physical systems. Dr Ramgoolam's previous work had shown that finite groups are instrumental in understanding the mathematical mechanisms of the AdS/CFT correspondence [9].

A key mathematical idea in string theory is that valuable physical insights emerge from studying how quantum field theories behave on a diverse range of two-dimensional surfaces. In a sequence of three papers [3-5], the researchers applied this idea to a characteristic fundamental to any finite group – a grid of mathematical data called the 'character table' of the group.

Within a finite group's character table, some integer quantities are easy to identify, while others are somewhat hidden. Through their research, the team realised that these traits reflect properties of measurable quantities in a certain class of string theories defined by combining information from physical systems (called lattice topological quantum field theories) formulated on surfaces of different shapes. This led them to identify a new duality, named 'row-column duality' of character tables. It relates the integers obtained by summing along the rows of a character table to those obtained by summing along its columns.

Another key mathematical component in string theory, particularly in gauge-string duality, is the notion of a graph (a collection of points and lines) embedded on a two-dimensional surface. These embedded graphs are called 'ribbon graphs'. They were first recognised by the famous Nobel-prize winning physicist Gerard 't Hooft in the 1970s as a key to understanding quantum field theories, in the limit where they contain very large numbers of particles [11].



Gerard 't Hooft, Dutch physicist and Nobel Prize winner.

It is precisely this kind of large particle limit in quantum field theories that features in the AdS/CFT correspondence. In their papers, Dr Ramgoolam and Dr Ben Geloun developed quantum mechanical models where the quantum states are combinations of ribbon graphs [1,2,6]. In this framework, they found new algorithms for computing group theoretic quantities, called 'Kronecker coefficients', which are of particular interest in complexity theory.

The quantum mechanical models based on ribbon graphs, as well as other such combinatorial structures, allow the formulation of new quantum algorithms associated with finite groups [2]. The complexity of these algorithms can be calculated using quantum computation techniques, and owing to the AdS/CFT correspondence, some of them have classical counterparts which researchers had not previously expected. This opens up new avenues for exploring the interface between AdS/CFT and complexity theory.

From complexity and algorithms to the challenges of string theory

Ultimately, Ramgoolam's team hopes that through a deeper mathematical understanding of complexity in quantum mechanics using string theory models, their approach will provide fresh perspectives on the relation between quantum mechanics and string theory. This in turn has the potential to unlock new ways of thinking about the challenge of the complex string theoretic landscape in theoretical physics.

Personal response

What are your personal motivations for studying model universes?

Any approach to physical reality using mathematical models involves focusing the models on interesting features which are expected to allow fruitful interaction between the mathematics and what is being modelled. The long-term task of string theory is very ambitious – to unify quantum physics and gravity while producing a good description of particle physics. It makes sense to simplify the string-theoretic models as far as possible when we are seeking to understand novel features of string theory such as gauge-string duality. Given the deep connections between string theory and mathematics, I feel compelled to explore to what extent dualities in string theory reflect dualities in mathematics and to what extent these insights into mathematics extend to a deeper understanding of new algorithms and their complexity in quantum physics. Simple model universes turn out to be a very good place to start in this quest.

How have the main goals of string theory changed over the past few decades?

The primary long-term goal of string theory remains the unification of quantum physics, gravity, and particle physics. Along the way, we have found surprises in the last few decades such as gauge-string duality and the AdS/CFT correspondence. My personal focus is currently to understand mathematical features of gauge-string duality more deeply and to uncover applications of these features to algorithms and complexity in quantum physics.

What are the most important outcomes of the research programme so far?

We have discovered that quantum mechanical systems defined using spaces of quantum states, which also have the mathematical structure of an algebra, are remarkably rich as a source of interesting quantum algorithms. String theory motivates the study of new complexity questions based on these algebraic quantum algorithms. One of these quantum mechanical systems, building on the mathematics of surfaces and graphs embedded on them, leads to new algorithms which realise mathematical quantities known as Kronecker coefficients, a classic object of study in mathematics. Gauge-string duality points to novel comparisons between quantum and classical algorithms based on the fact that the same task has different realisations on the two sides of a gauge-string duality. The row-column duality of character tables which can be formulated in terms of two-dimensional combinatorial string theories is another significant highlight with rich potential for further developments.

Details

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Bio

Dr Sanjaye Ramgoolam completed his PhD in theoretical physics at Yale University. He held postdoctoral positions at Princeton and Brown Universities. He is a Reader at Queen Mary University of London, and currently Visiting Professor at the Dublin Institute for Advanced Studies and the Institute for Advanced Studies at Soochow University.

Further reading

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