



Capturing the mathematics of complex, evolving systems

Details



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Bio

Francisco Jurado is Professor-Researcher at Tecnológico Nacional de México (TecNM)/Instituto Tecnológico de La Laguna, Torreón, México. He received his DSc degree from Centro de Investigación y de Estudios Avanzados (CINVESTAV) del Instituto Politécnico Nacional (IPN) Unidad Guadalajara (México). He is a member of the CONACYT's National System of Researchers (SNI) at Level 1. Jurado has been distinguished as State Honorific Researcher by the Government of the State of Coahuila de Zaragoza, Mexico,

and the Science and Technology State Council (COEYCT).

Further reading

Ramírez, AA, & Jurado, F, (2023) The regulator problem to the convection-diffusion equation. *Mathematics*, 11, 1944. doi.org/10.3390/math11081944

Jurado, F, & Ramírez, AA, (2020) State feedback regulation problem to the reaction-diffusion equation. *Mathematics*, 8(11), 1–16. doi.org/10.3390/math8111983



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Capturing the mathematics of complex, evolving systems

- Like smoke particles rising from a fire, the motion of the particles is driven by both convection and diffusion.
- Describing how the particles move requires a collection of partial differential equations called a distributed parameter system.
- Professor Francisco Jurado at the Tecnológico Nacional de México has developed and tested a new mathematical model for describing the convection and diffusion effects.
- His work has far-reaching applications, from traffic flow control, air pollution, and drug delivery for patients.

Many everyday phenomena require very complex mathematics and physics to describe how they happen. For example, describing how smoke particles rise and move from a fire requires an understanding of both convection and diffusion physics. But being able to predict the movements of smoke particles and how they spread is very important in fire safety – smoke inhalation is one of the most common causes of death from fire, whether that is a small-scale local house fire or bigger wildfires that can affect huge areas.

Modelling and predicting smoke particle movement relies on being able to find a series of equations that describe the smoke particle motion accurately and being able to solve them to see how the system of smoke particles evolves in time. This concept is at the heart of all mathematical modelling and good quality mathematical models can help us not just understand historical behaviour, ie, why did the smoke spread the way it did, but also to predict where the smoke particles will spread next. Many safety precautions rely on these types of predictions to improve building design, for example, to minimise smoke spread.

The challenge for researchers is to develop the often complicated series of equations that are needed to describe these phenomena

and ensure that they can be solved to recover information on the location of the objects over time. Often the systems of equations needed to describe such phenomena are based on partial differential equations: the series of equations that describe the location and time-evolution of a system are known as a distributed parameter system.

Professor Francisco Jurado at the Tecnológico Nacional de México has been working on approaches to solve the problem of distributed parameter systems to describe diffusion-convection systems. He has recently developed an approach using a combination of approaches, including the Sturm-Liouville differential operator and the regulator problem, to develop a model for diffusion-convection behaviour that is sufficiently stable and free of external disturbances. Importantly, this approach allows us to yield meaningful information for real systems.

Mathematical models can help us not just understand historical behaviour but predict where the smoke particles will spread next.





Jurado's research has many applications in the real world, including traffic flow control, gas and oil extraction, drug delivery, and air pollution.

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Convection and diffusion

Convection is how heat is transferred between fluids, such as liquids and gases, and describes the process of how the fluid moves from a region of higher heat energy to lower energy. One example of natural convection in fluids is that warm air rises because it is less dense than cold air so warm air has an increased buoyancy.

Light particles, like smoke, are transported by the fluid they are present in, so when the hot air rises, this causes the particles to rise as well. However, convection is not the only process that affects the movement of particles with time.

Diffusion is the process where a substance will move from a region of higher concentration to lower concentration. When smoke particles are produced by a fire, the concentration of particles will be highest at the source of the fire and so the particles will naturally start to spread over time to regions of lower concentration.

Jurado and his colleague are working with a series of equations to describe the combined movement from diffusion and convection using partial differential equations. One of the challenges they have faced is that the types of system that can be described by convection–diffusion equations are infinite-dimensional systems. Essentially, they possess infinite degrees of freedom and can travel anywhere in any configuration.

In practical terms, that means Jurado needed to find functions to describe the behaviour of the system rather than a finite number of coordinates in time. An infinite-dimensional system, even with very advanced high-powered computers, is also impossible to solve without reasonable boundary conditions that limit the behaviour of the system to something physical ie, the fluids cannot move faster than certain speeds.

Francis equations

Infinite dimensional systems need particular mathematical analysis techniques to be solved. While the original physical equations to describe the physics of diffusion and convection were known, Jurado looked for a 'stable' solution for the full, complex control system.

Jurado achieved this through the Francis equations – a type of regulator equation. Regulator equations check that the output of the control system (the series of partial differential equations that make the mathematical model) is reasonable and consistent with the correct physics. Often these complex series of equations must be solved numerically, and unrealistic values can arise for the solution for a single time point.

The regulator equations can check the validity of a given solution and reject it if it is not appropriate; this is known as disturbance rejection. The equations can then be re-solved with a slightly modified set of input values. The simulation can then continue without further problems, maintaining realistic physical behaviour.

Wide applications

Having advanced mathematical tools, such as those provided by Jurado, is an important development in tackling complex real-world challenges. This robust system of equations and distributed parameter system is not just applicable to phenomena like the transport and movement of smoke particles. Jurado's research will have many far-reaching implications. Convection–diffusion equations are used in many areas of life, including describing how pollutants move in the environment, whether that is air or in water, in describing how medicines are transported in the human body, or how materials can be corroded by contaminants.

Personal response

What is your approach and inspiration when tackling these kinds of mathematical problems?

When dealing with these kinds of problems, the first thing that comes to my mind is a review of the mathematical tools that I have at hand to tackling the problem in question. The study of the state-of-the-art plays the essential part of the work since we can detect the most recent contributions and proposals about the methodologies oriented to the control of distributed parameter systems. Then, we explore the principles on which these approaches lie and go deep on their study. Our inspiration comes from the desire to solve the problem at hand, whether through a novel proposal or an alternative way that will make the problem solution a feasible one. Or, in contrast, to detect the obstacles and methodologies that do not help in the searching for the solution.

How would you like to see your research applied to real-world challenges?

This is an interesting question since experimental platforms are not easy to implement due to that we are trying with distributed parameter systems, that is infinite-dimensional systems, which implies a great investment because these are very complex systems. In fact, the strategical allocation of sensors and actuators inside the whole system is not easy, if not impossible, to be carried out. There is so much to study and to experiment about these phenomena. As for most of the scientific community members, the experimental evaluation is desirable to get a validation and better comprehension of the models/phenomena under study, so it would give me great satisfaction to see the results of our proposal experimentally validated in order to evaluate its viability when confronting real-world problems.



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